QUEST– USER MANUAL

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Part 1. Introduction

1. About QUEST

QUEST is an open source software tool (available at http://www.hcs.ei.tum.de/software) for automated controller synthesis of incrementally input-to-state stable nonlinear control systems. The tool is implemented in C++ and contains two major parts:

1. Construction of symbolic abstractions: the tool uses state-space quantization-free approach for the construction of symbolic abstractions which can be potentially more beneficial for systems with high-dimensional state-space.
2. Symbolic controller synthesis: the synthesis of controller is implemented using fixed point computations.

The implementation of QUEST uses *binary decision diagrams* (BDD) [1] as an underlying data structure for memory-efficient storage and computation of symbolic abstractions and controllers. Operations on BDDs are handled with the help of CUDD binary decision diagram library.

The tool is intended to be used and extended by researches in the area of formal synthesis of complex systems.

In this part, we give a quick overview of the basic concepts which will be used throughout the manual. We also briefly review the theory behind the tool.

2. Symbolic Controller Synthesis

2.1. **Control System.** QUEST supports computation of symbolic controllers for incrementally input-to-state stable nonlinear control systems of the form

$$\dot{\xi} = f(\xi, \upsilon), \tag{1}$$

where f is given by $f : \mathbb{R}^n \times \mathsf{U} \to \mathbb{R}^n$ and $\mathsf{U} \subseteq \mathbb{R}^m$ is a bounded set. We assume that $f(., u)$ is continuously differentiable for every $u \in \mathsf{U}$. Let $\xi_{x,\upsilon}(t)$ denote the solution of (1) starting from initial condition x under input signal $\upsilon \in \mathcal{U}$ at time $t \geq 0$, where $\mathcal{U}$ denotes the set of all measurable input signals. For the sake of completeness, we recall the definition of incremental input-to-state stability [2]:

Definition 2.1. *A nonlinear control system as in (1) is said to be incrementally input-to-state stable if there exist a $\mathcal{KL}$ function β and a $\mathcal{K}_\infty$ function γ such that for any $t \geq 0$, any two initial conditions x and $\hat{x}$, and any $\upsilon, \hat{\upsilon} \in \mathcal{U}$, the following condition is satisfied:*

$$\|\xi_{x,\upsilon}(t) - \xi_{\hat{x},\hat{\upsilon}'}(t))\| \leq \beta(\|x - \hat{x}\|, t) + \gamma(\|\upsilon - \upsilon'\|_\infty).$$

We refer the interested reader to [2] for definitions of $\mathcal{KL}$ and $\mathcal{K}_\infty$ functions and other preliminaries.

2.2. **A Unified Framework for Control Systems and Symbolic Abstractions.** We present a notion of system [3] which serves as a unified modelling framework for nonlinear control systems in (1) and their symbolic abstractions. A system is a tuple

$$S = (X, X_0, U, \longrightarrow, Y, H),$$

consisting of: a (possibly infinite) set of states X; a (possibly infinite) set of initial states $X_0 \subseteq X$; a (possibly infinite) set of inputs U; a transition relation $\longrightarrow \subseteq X \times U \times X$; a set of outputs Y; and an output map $H : X \to Y$.

We use notation $x \xrightarrow{u} x'$ for a transition $(x, u, x') \in \longrightarrow$, where state x' is a u-successor (or simply successor) of state x, for some input $u \in U$. System S is called *metric* if the output set Y is equipped with a metric $\mathbf{d} : Y \times Y \longleftrightarrow \mathbb{R}_0^+$.

2.3. **Approximate Bisimulation Relation.** We introduce the notion of approximate bisimulation relation [4] which is used to relate original systems (1) with their symbolic abstractions.

Definition 2.2. *Let $S_1 = (X_1, X_{10}, U_1, \underset{1}{\longrightarrow}, Y_1, H_1)$ and $S_2 = (X_2, X_{20}, U_2, \underset{2}{\longrightarrow}, Y_2, H_2)$ be two metric systems with the same output sets $Y_1 = Y_2$ and metric $\mathbf{d}$. For $\varepsilon \geq 0$, a relation $\mathcal{R} \subseteq X_1 \times X_2$ is said to be an ε-approximate bisimulation relation between S_1 and S_2 if it satisfies following conditions:*

(i) $\forall (x_1, x_2) \in \mathcal{R}$, *we have* $\mathbf{d}(H_1(x_1), H_2(x_2)) \leq \varepsilon$;
(ii) $\forall (x_1, x_2) \in \mathcal{R}$, $x_1 \underset{1}{\xrightarrow{u_1}} x_1'$ *in* S_1 *implies* $x_2 \underset{2}{\xrightarrow{u_2}} x_2'$ *in* S_2 *satisfying* $(x_1', x_2') \in \mathcal{R}$;
(iii) $\forall (x_1, x_2) \in \mathcal{R}$, $x_2 \underset{2}{\xrightarrow{u_2}} x_2'$ *in* S_2 *implies* $x_1 \underset{1}{\xrightarrow{u_1}} x_1'$ *in* S_1 *satisfying* $(x_1', x_2') \in \mathcal{R}$.

2.4. **Computation of Symbolic Abstractions.** We consider the sampled behavior of (1) with sampling time $\tau > 0$. Then the corresponding system representation of (1) is given by the tuple

$$S_1 = (X_1, X_{10}, U_1, \underset{1}{\longrightarrow}, Y_1, H_1),$$

where $X_1 = \mathbb{R}^n$, $X_{10} \subseteq X_1$, $U_1 = \mathcal{U}_\tau$, $\mathcal{U}_\tau := \{\upsilon \in \mathcal{U} \mid \upsilon(t) = \upsilon(k\tau), t \in [k\tau, (k+1)\tau[, k \in \mathbb{N}\}$, there exists a transition $x_1 \underset{1}{\xrightarrow{u}} x_1'$ iff $x_1' = \xi_{x_1,u}(\tau)$, $Y_1 = X_1$, $H = 1_x$.

QUEST computes symbolic models that are related via approximate bisimulation relation to S_1. For constructing a symbolic abstraction of S_1 we use a state-space quantization-free approach as discussed in details in [5, 6, 7]. We assume that the control system in (1) is incrementally input-to-state stable. Under this assumption, one can construct an approximate bisimilar symbolic abstraction of S_1. Let $\overline{U}$ denote the set of quantized inputs corresponding to U with a quantization parameter η, N be the temporal horizon, and x_s be a source state, then the symbolic abstraction of S_1 is given by a tuple

$$S_2 = (X_2, X_{20}, U_2, \underset{2}{\longrightarrow}, Y_2, H_2),$$

where

- $X_2 = \overline{U}^N$, $X_{20} = X_2$, $U_2 = \overline{U}$, $Y_2 = Y_1$;
- $x_2 \underset{\rho}{\xrightarrow{u}} x_2'$, where $x_2 = (u_1, u_2, \ldots, u_N) \in X_2$, if and only if $x_2' = (u_2, \ldots, u_N, u)$ for some $u \in U_2$;
- $H_2(x_2) = \xi_{x_s,x_2}(N\tau)$.

Here, we abused notation by identifying $x_2 = (u_1, u_2, \ldots, u_N) \in \overline{U}^N$ with the input curve υ with domain $[0, N\tau[$ such that $\upsilon(t) = u_k$ for any $t \in [(k-1)\tau, k\tau[$ for $k \in \{1, \ldots, N\}$.
For more details on the construction of symbolic abstractions using state-space quantization-free approach, the interested readers are referred to the results in [5, 7, 6].

2.5. **Controller Synthesis via Fixed Point Computation.** QUEST natively supports invariance (often referred to as safety) and reachability specifications. For the synthesis of controllers C enforcing these specifications, we make use of two fixed point algorithms: minimum fixed point and maximum fixed point algorithm. Moreover, QUEST also supports customized specifications such as reach and stay by combining these two algorithms. The controller synthesis using fixed point computation is similar to the one used in SCOTS [8]; see https://www.hcs.ei.tum.de/en/software/scots/ for more details.

Part 2. Getting Started with QUEST

3. Source Code Organization

```
./manual /* manual of QUEST */
./src /* the source code of the QUEST */
./examples /* directory containing various examples*/
./license.txt /* the license file */
./readme.txt /* a quick description pointing to this manual */
./installation_notes_windows. txt /* installation guidlines for windows platform*/
```

4. Installation

In general, QUEST is implemented in "header-only" style and you only need a working C++ developer environment. However, QUEST uses the CUDD library maintained by Fabio Somenzi, which can be downloaded at http://vlsi.colorado.edu/~fabio/.

The requirements and installation instructions are summarized as follows:

(1) A working C/C++ development environment
- Mac OS X: You should install Xcode.app including the command line tools
- Linux: Most linux OS includes the necessary tools already
- Windows: You need to have MSYS-2 installed or use the latest update of Windows 7 providing support for Ubunto-on-windows.

(2) A working installation of the CUDD library and enabling the following options
- the C++ object-oriented wrapper,
- the dddmp library, and
- the shared library.

The package follows the usual `configure`, `make`, and `make install` installation routine. We use `cudd-3.0.0`, with the following configuration

```
$ ./configure --enable-shared --enable-obj --enable-dddmp
--prefix=/opt/local/
```

On Windows and linux, we experienced that the header files `util.h` and `config.h` were missing in `/opt/local` and we manually copied them to `/opt/local/include`.
For further details about windows installations (which is somehow different), please refer to the `installation_notes_windows.txt` file within QUEST. You should also test the BDD installation by compiling a dummy program, e.g. `test.cc` as the following

```
#include<iostream>
#include "cuddObj.hh"
#include "dddmp.h"
int main () {
Cudd mgr(0,0);
BDD x = mgr.bddVar();
}
```

which should be compiled using

```
$ g++ test.cc -I/opt/local/include -L/opt/local/lib -lcudd
```

5. Running a Sample Example

For a quick start

(1) go to one of the examples in

```
./examples/trafficmodel /* five dimensional traffic model for safety specification */
./examples/thermalmodel10R /* ten-room thermal model for safety specification */
./examples/thermalmodel6R /* six-room thermal model for reach and stay specification */
```

(2) read the readme file (if the directory contains one)
(3) edit the `Makefile` file
   (a) adjust the used compiler
   (b) adjust the directories of the CUDD library
(4) compile and run the executable, for example in `/examples/trafficmodel` run

```
$ make
$ ./trafficmodel
```

(5) for graphical visualization of output, run the m file, for example in `./examples/trafficmodel` run

```
>> trafficmodel.m
```

(6) modify the example to your needs.

6. Implementation of QUEST

In this section, we describe the architecture of QUEST. The algorithm is mainly distributed among three C++ classes:

- `SymbolicSetSpace`
- `getAbstraction`
- `fixedPointMode`

6.1. `SymbolicSetSpace`. The `SymbolicSetSpace` is the main class in which the transition relations as described in section 2.4 are computed with the help of binary decision diagrams (BDDs) [1] as underlying data structure. Specifically, we use the object oriented wrapper in the CUDD library [9]. It accepts temporal horizon N, dimension of input space $iDIM$, lower bound on input space lb, upper bound on input space ub, and quantization parameter η. The class `SymbolicSetSpace` directly constructs the transition relations as

Algorithm 1 Computation of transition relation

Require: N, lb, ub, η, $iDIM$
1: **for** $i = 1$ to $iDIM$ **do**
2: number of elements in quantized Input[i]= $(ub[i] - lb[i])/\eta[i]$
3: Number of states in abstraction= $(\Pi_{i=1}^{iDIM}$ number of elements in quantized Input[i]$)^N$
4: Let $x = (u_1, u_2, \ldots, u_N)$ be a state in abstraction, $x' = (u'_1, u'_2, \ldots, u'_N)$, and u be an input
5: **for all** x and u **do**
6: **for** $i = 1$ to $N - 1$ **do**
7: $u'_i = u_{i+1}$
8: $u'_N = u$

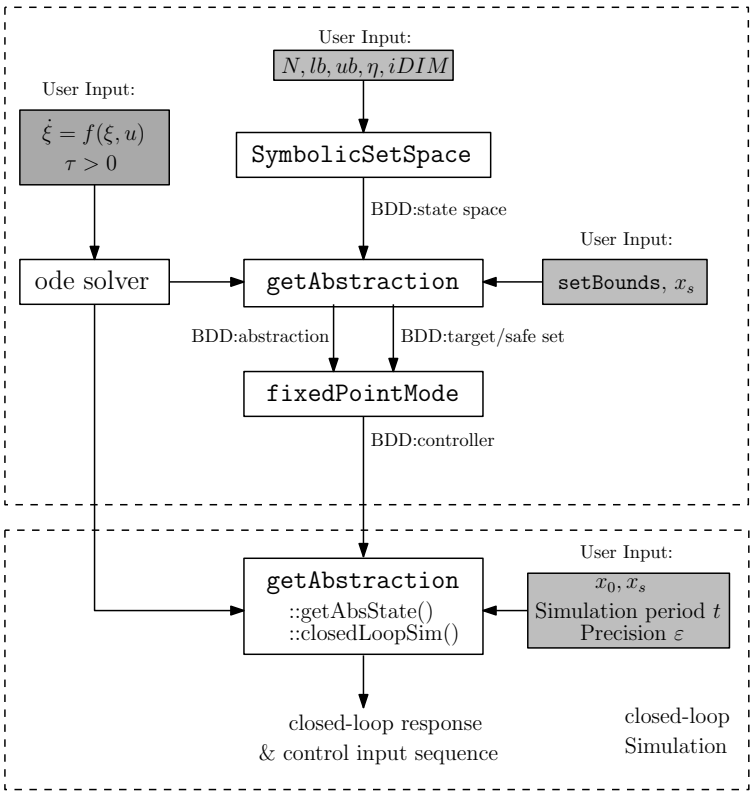

FIGURE 1. Workflow.

6.2. `getAbstraction`. The `getAbstraction` is a derived class of the `abstractionMode` which manages all BDD related information, such as number and indices of variables. The `getAbstraction` class provides some supporting functions that are required for overall operation of `QUEST`. Some of important functions are listed below:

```
getAbstraction::getAbstractSet() /* get set of abstract states whose outputs are in safety/target region */
getAbstraction::getOutput() /* get output H(x) corresponding to state x in the abstraction*/
getAbstraction::getAbsState() /* get abstract state related to concrete state in the original system*/
getAbstraction::closedLoopSim() /* closed-loop simulation and printing output response */
```

6.3. `fixedPointMode`. This class implements fixed point computation for the synthesis of controller. In particular, we use the methods `fixedPointMode::reach()`, `fixedPointMode::safe()` and `fixedPointMode::reachStay()` to synthesize controllers by solving fixed point computation for reachability, safety, and reach and stay specification, respectively.
The general work flow explaining use of classes with the different user inputs and the possible tool output is illustrated in Figure 1.

7. USAGE OF `QUEST`

In order to use `QUEST`, we create a C++ file in which we include the cudd library `cuddObj.hh` and header-only classes `SymbolicSetSpace.hh, abstractionMode.hh, getAbstraction.hh,` and `fixedPointMode.hh`. We begin with defining temporal horizon N, sampling time T, and the dynamics of the system. For the synthesis of controller, the solution of (1) is required for which we require to work with numerical approximations obtained by a numerical ODE solver. For example, in the

thermal model example in `./examples/thermalmodel6R` a fixed step size Runge Kutte scheme of order four has been used. For implementation we use:

```
const double T = 25; /* Sampling time */
const size_t N = 6; /* Temporal Horizon */
const int sDIM = 6; /* System dimension */
const int iDIM = 2; /* Input dimension */
typedef std::array<double,sDIM> state_type; /* state type */
auto system_post = [](state_type &x, double* u) -> void {
/* ode describing six-room thermal model*/
auto rhs=[us](state_type &xx, const state_type &x) -> void {
const double a=0.05;
const double ae1=0.005;
const double ae4=0.005;
const double ae=0.0033;
const double ah=0.0036;
const double te=10;
const double th=100;
xx[0] = (-3*a-ae1-ah*us[0])*x[0]+a*x[1]+a*x[2]+a*x[4]+ae1*te+ah*th*us[0];
xx[1] = (-2*a-ae)*x[1]+a*x[0]+a*x[3]+ae*te;
xx[2] = (-2*a-ae)*x[2]+a*x[0]+a*x[3]+ae*te;
xx[3] = (-3*a-ae4-ah*us[1])*x[3]+a*x[1]+a*x[2]+a*x[5]+ae4*te+ah*th*us[1];
xx[4] = (-a-ae)*x[4]+a*x[0]+ae*te;
xx[5] = (-a-ae)*x[5]+a*x[3]+ae*te;
};
size_t nint = 5; /* no. of time step for ode solving */
double h=T/nint; /* time step for ode solving (T is the sampling time) */
ode_solver(rhs,x,nint,h); /* Runga Kutte solver */
}
```

Subsequently, we define a function `setBound` which is used to find set of abstract states whose outputs $H(x)$ are within the safety/target region defined over the set of states of concrete system. The safe set used in thermalmodel6R example is given as:

```
/* defining safe set for the controller
ul[i] : upper bound on safe region for the temperature in the ith room
ll[i] : lower bound on safe region for the temperature in the ith room */
auto setBound = [](state_type y) -> bool {
  double ul=21.0;
  double ll=17.5;
  bool s = true;
  for(int j = 0; j < sDIM; j++){
    if( y[j] >= ul || y[j] <= ll ){
      s = false;
      break;
    }
  }
  return s;
}
```

The user also needs to provide information about system inputs. `QUEST` supports two types of inputs: i) continuous, bounded, input with lower and upper bound with quantization parameter and ii) discrete set of inputs. The example thermalmodel6R uses continuous bounded inputs as shown below:

```
  /* lower bounds on inputs */
     double lb[sDIM]={0,0};
  /* upper bounds on inputs */
     double ub[sDIM]={1,1};
  /* quantization parameter */
```

```
double eta[sDIM]={0.5,1};
```

and the example thermalmodel10R considers discrete input set as

```
const size_t P = 3; /* Number of elements in the input set*/
double ud[P][iDIM]={{0,0},{0,1},{1,0}};
```

Now to implement the construction of symbolic abstraction as discussed in Section 2.4 using class `SymbolicSetSpace`. We use for discrete inputs:

```
SymbolicSetSpace ss(ddmgr,P,N); /*for discrete input set*/
ss.addAbsStates();
```

and for continuous inputs:

```
SymbolicSetSpace ss(ddmgr,iDIM,lb,ub,eta,N); /*for continuous input set*/
ss.addAbsStates();
```

Further, we obtain the set of abstract states whose outputs are inside the safe/target region by using `getAbstractSet` in class `getAbstraction` to implement the following algorithm:

Algorithm 2 Constrain set computation

Require: x_s, `system_post`, `setBounds`

1: **for all** $x \xrightarrow{u} x'$ **do**
2: **if** $\xi_{x_s,x}(NT)$ obtained using `system_post` satisfies `setBounds` **then**
3: add corresponding BDD state to `BDD set`
4: **else**
5: discard it

and is used as

```
/* defining abstraction class */
getAbstraction<state_type> ab(&ss);
/* Computing the set of abstract state satisfying setBounds */
BDD set = ab.getAbstractSet(system_post,setBounds,xs);
```

The controller synthesis is carried out using fixed point computation using `class fixedPointMode` whose source code can be found in `./src/FixedPointMode.hh`. In particular, we use methods `FixedPointMode::safe`, `FixedPointMode::reach`, and `FixedPointMode::reachStay` to compute controllers enforcing safety, reachability, and reach and stay, respectively. In thermalmodel6R example, we instantiate an object of `FixedPointMode` with the symbolic model S_2 given by a `SymbolicSetSpace` object `ab`

```
fixedPointMode fp(&ab);
```

and controller for reach and stay specification is synthesized using

```
BDD C;
/* controller for reach and stay specification */
C = fp.reachStay(set);
```

To run a closed-loop simulation with the synthesized controller, we need to find an initial state in the abstraction whose output is ε-close to the concrete initial state x_0. We use the following code to compute initial abstract state and execute the closed-loop simulation.

```
/* finding initial abstract state in the abstraction*/
BDD w0 = ab.getAbsState(system_post,x0,xs,epsilon,sDIM);
/* closed-loop simulation for time period t*/
ab.closedLoopSim(C,w0,system_post,x0,sDIM,t);
```

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