CommonRoad: Cost Function Specification

(Version 2018b)

Stefanie Manzinger, Markus Koschi, and Matthias Althoff

Technische Universität München, 85748 Garching, Germany

Abstract

Motion planning for a single vehicle or multiple vehicles usually requires the minimization of some objectives, e.g., acceleration and fuel consumption. In order to facilitate the reproducibility of results for motion planning, this document describes various standardized cost functions as part of the CommonRoad benchmark suite. We present how various partial cost functions can be combined to a new optimization objective and introduce a compact notation to unambiguously define the applied cost function.

Contents

1 Changes Compared to Version 2018a

The vehicle models did not change compared to version 2018a.

2 Motion Planning

We introduce $f_M(x(t), u(t))$ as the right-hand side of a state space model of vehicle model M:

$$\dot{x}(t) = f_M(x(t), u(t)), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$ is the input vector. The available vehicle models M of the CommonRoad benchmark suite are defined in *CommonRoad: Vehicle Models*. We denote the initial state provided by scenario S as $x_{0,S} \in \mathbb{R}^n$, the initial time as t_0 and the final time as t_f.

A possible solution of the motion planning problem requires that hard constraints are satisfied: the ego vehicle has to reach a goal region $\mathcal{G}_S \subset \mathbb{R}^n$ without causing a collision with obstacles, e.g., other traffic participants. Thus, the occupancy of the vehicle $O(x(t))$ must be in the free space $\mathcal{W}_{\mathtt{S,free}}(t) \subset \mathbb{R}^2$ for all $t \in [t_0, t_f]$. Moreover, constraints $g_S(x(t), u(t), t) \leq 0$, such as speed limits or other traffic rules [5], must be obeyed.

In addition to hard constraints, there exist objectives codifying conditions which are desirable but no necessity, e.g., minimization of engine power, which are included in the cost function J_C with ID C:

$$J_C(x(t), u(t), t_0, t_f) = \Phi_C(x(t_0), t_0, x(t_f), t_f) + \int_{t_0}^{t_f} L_C(x(t), u(t), t)\, \mathtt{d}t,$$

with input $u(t)$, terminal cost Φ_C, and running cost L_C.

Summarizing, the motion planning problem is

$$u^*(\cdot) = \arg\min_{u(\cdot)} J_C(x(t), u(t), t_0, t_f)$$

subject to

$$\begin{aligned} &\dot{x}(t) = f_M(x(t), u(t)), \quad O(x(t)) \in \mathcal{W}_{\mathtt{S,free}}(t), \\ &g_S(x(t), u(t), t) \leq 0, \quad x(t_0) = x_{0,S}, \quad x(t_f) \in \mathcal{G}_S. \end{aligned}$$

Analogously to the composability of the benchmarks, we express the overall cost function of ID C as the weighted sum of partial cost functions. The partial cost functions J_p have an unique ID p and the set $\mathcal{P}$ contains all IDs of the proposed partial cost functions. The overall cost function is obtained by the weighted sum

$$J_C(x(t), u(t), t_0, t_f) = \sum_{i \in \mathcal{I}} w_i\, J_i(x(t), u(t), t_0, t_f),$$

where $\mathcal{I} \subset \mathcal{P}$ contains the IDs of the applied partial cost functions and $w_i \in \mathbb{R}^+$ are weights. In the subsequent section, we present the partial cost functions, which are available in the CommonRoad benchmark suite.

3 Cost Functions

This section shows the standardized cost functions available in CommonRoad, which are categorized into *running costs* and *terminal costs*.

3.1 Running Costs

- **Acceleration**: $J_A = \int_{t_0}^{t_f} a^2\, \mathtt{d}t$, with acceleration a (see e.g. [8, Sec. III.B]).
- **Jerk**: $J_J = \int_{t_0}^{t_f} \dot{a}^2\, \mathtt{d}t$, with jerk $\dot{a}$ (see e.g. [6, Sec. III]).
- **Steering angle**: $J_{SA} = \int_{t_0}^{t_f} \delta^2\, \mathtt{d}t$, with steering angle δ (see e.g. [4]).
- **Steering rate**: $J_{SR} = \int_{t_0}^{t_f} v_\delta^2\, \mathtt{d}t$, with steering velocity v_δ (see e.g. [4]).
- **Energy**: $J_E = \int_{t_0}^{t_f} P(x, u)\, \mathtt{d}t$, where $P(x, u)$ is the required power of the engine for the state x and the input u, which can be obtained from engine mappings (see e.g. [3, Sec. III.B]).

- **Yaw rate**: $J_Y = \int_{t_0}^{t_f} \dot{\Psi}^2 \, \mathtt{d}t$, with yaw rate $\dot{\Psi}$ (see e.g. [8, Sec. III.B]).
- **Lane center offset**: $J_{LC} = \int_{t_0}^{t_f} d^2(t) \, \mathtt{d}t$, where $d(t)$ is the distance to the lane center or a driving corridor (see e.g. [8, Sec. III.B]).
- **Velocity offset**: $J_V = \int_{t_0}^{t_f} (v_{\mathtt{des}}(x(t)) - v(t))^2 \, \mathtt{d}t$, where $v_{\mathtt{des}}(x(t))$ is the desired velocity for the vehicle state x and $v(t)$ is the current velocity of the vehicle (see e.g. [8, Sec. III.B]).
- **Orientation offset**: $J_O = \int_{t_0}^{t_f} (\theta_{\mathtt{des}}(x(t)) - \theta(t))^2 \, \mathtt{d}t$, where $\theta_{\mathtt{des}}(x(t))$ is the desired orientation for the vehicle state x and $\theta(t)$ is the current orientation of the vehicle (see e.g. [4]).
- **Distance to obstacles**: $J_D = \int_{t_0}^{t_f} \max(\xi_1, \dots, \xi_o) \, \mathtt{d}t$, where o is the number of surrounding obstacles, $\xi_i = e^{-w_{\mathtt{dist}} d_i}$, d_i is the distance of the ego vehicle to an obstacle, and $w_{\mathtt{dist}}$ is an additional required weight (see e.g. [7, eq. 7-8]).
- **Path length**: $J_L = \int_{t_0}^{t_f} v \, \mathtt{d}t$, with velocity v (see e.g. [7, Tab. 1]).
- **Inverse Duration**: $J_{ID} = \frac{1.0}{\tau}$, where τ is the duration of the planned trajectory.

3.2 Terminal Costs

- **Time**: $J_T = t_f$ (see e.g. [2, eq. 2]).

4 Notation

Let us now introduce a notation for writing the used weights compactly. We write $w_T = 0.1$, $w_{SA} = 0.4$, and $w_Y = 0.7$ in short as $[(T|0.1), (SA|0.4), (Y|0.7)]$. After agreeing that we use SI units for all variables, this notation uniquely defines a cost function. Most works, however, do not provide such weights, so we cannot include their values in the current version of the benchmark. We therefore hope that once the structure is fixed, other researchers will contribute their used cost functions and weights. Works that published their used weights are listed in Tab. 1, where the cost function ID is chosen as the initials of the first authors plus a running number.

We further introduce the superscript notation $\square^{\text{lon}}$ and $\square^{\text{lat}}$ for partial cost terms that only concern longitudinal or lateral planning direction. E.g. $(A^{\text{lat}}|0.5)$ corresponds to $0.5 * \int_{t_0}^{t_f} a_{\text{lat}}^2 \, \mathtt{d}t$, where a_{lat} means the lateral acceleration.

5 Conclusions

The usage of standardized cost functions in the CommonRoad benchmark suite facilitates the precise description of numerical experiments. In order to unambiguously define the applied cost function, only a short ID and the corresponding weights have to be stated. In the future, the cost functions can be extended by other researchers and will also be extended by ourselves.

Acknowledgment

The author gratefully acknowledge financial support by the Deutsche Forschungsgemeinschaft (German Research Foundation) within the Priority Program SPP 1835 "Cooperative Interacting

Table 1

ID C	Partial cost functions and weights	Reference
JB1	$[(T\|1)]$	[2, eq. 2]
SA1	$[(SA\|0.1), (SR\|0.1), (D\|10^5)]$	Inspired by [1, eq. 2]
WX1	$[(T\|10), (V\|1), (A\|0.1), (J\|0.1), (D\|0.1), (LC\|10)]$, $w_{\mathtt{dist}} = 1$	Inspired by [7, Tab. IV]
SM1	$[(A\|50), (SA\|50), (SR\|50), (LC\|1), (V\|20), (O\|50)]$	Combination of eq. 8 and eq. 9 in [4]
SM2	$[(A\|50), (SA\|50), (SR\|50), (LC\|1), (O\|50)]$	[4, eq. 8]
SM3	$[(A\|50), (SA\|50), (SR\|50), (V\|20), (O\|50)]$	[4, eq. 9]
MW1	$[(J^{\text{lat}}\|5.0), (J^{\text{lon}}\|0.5), (V^{\text{lon}}\|0.2), (ID\|1.0)]$	Inspired by [6, eq. 2 and sec. 5.B]

Automobiles" (grant number: AL 1185/4-1) and by the BMW Group within the Car@TUM project.

References

[1] S. J. Anderson and S. C. Peters. An optimal-control-based framework for trajectory planning, threat assessment, and semi-autonomous control of passenger vehicles in hazard avoidance scenarios. *International Journal of Vehicle Autonomous Systems*, 8:190–216, 2010.

[2] J. E. Bobrow. Optimal robot path planning using the minimum-time criterion. *IEEE Journal on Robotics and Automation*, 4(4):443–450, 1988.

[3] D. Kim, H. Peng, S. Bai, and J. M. Maguire. Control of integrated powertrain with electronic throttle and automatic transmission. *IEEE Transactions on Control Systems Technology*, 15(3):474–482, 2007.

[4] S. Magdici and M. Althoff. Fail-safe motion planning of autonomous vehicles. In *Proc. of the 19th IEEE International Conference on Intelligent Transportation Systems*, pages 452—458, 2016.

[5] A. Rizaldi and M. Althoff. Formalising traffic rules for accountability of autonomous vehicles. In *Proc. of the 18th IEEE International Conference on Intelligent Transportation Systems*, pages 1658–1665, 2015.

[6] M. Werling, J. Ziegler, S. Kammel, and S. Thrun. Optimal trajectory generation for dynamic street scenarios in a Frenét Frame. In *Proc. of the IEEE International Conference on Robotics and Automation*, pages 987–993, 2010.

[7] W. Xu, J. Wei, J. M. Dolan, H. Zhao, and H. Zha. A real-time motion planner with trajectory optimization for autonomous vehicles. In *Proc. of the IEEE International Conference on Robotics and Automation*, pages 2061–2067, 2012.

[8] J. Ziegler, P. Bender, T. Dang, and C. Stiller. Trajectory planning for BERTHA – a local, continuous method. In *Proc. of the IEEE Intelligent Vehicles Symposium*, pages 450–457, 2014.